Tuesday, March 16, 2021 1:43 PM

Indeterminate forms

$$\lim_{x \to 0} \frac{x^2+2x}{x+1} = ?$$

Put $x=0$

$$= \frac{0+0}{0+1} = 0 \to \text{finite} \to \frac{0}{0}$$

While solving limits, we can have forms

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, ∞^0, 0^0, 1^∞, $\infty - \infty$

are called as indeterminate forms

L'Hospital's Rule for $\frac{0}{0}, \frac{\infty}{\infty}$

$$\lim_{x \to c} \frac{F(x)}{g(x)} = \lim_{x \to c} \frac{F'(x)}{g'(x)} = \lim_{x \to c} \frac{F''(x)}{g''(x)}$$

Que: $\lim_{x \to 0} \frac{\sin x}{x} = ?$

Put $x=0$, $\frac{0}{0}$ form, Apply L'Hospital Rule

Put $x=0$, $\frac{0}{0}$ form, Apply L Hospital Rule

$$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{\cos x}{1} = \cos 0 = 1$$

Que $\lim_{x\to 0}\frac{\tan x}{x} = ?$ $\left(\frac{0}{0}\text{ form}\right)$

By L Hos Rule

$$\lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{\sec^2 x}{1} = 1$$

Que $\lim_{x\to 0}\frac{\sin ax}{\sin bx} = ?$

$\frac{0}{0}$ form, By L-H Rule

$$\lim_{x\to 0}\frac{\cos ax \cdot a}{\cos bx \cdot b} = \frac{a}{b}$$

Que $\lim_{x\to 0}\frac{(x-\tan x)}{x-\sin x}$ $\left(\frac{0}{0}\text{ form}\right)$

$$= \lim_{x\to 0}\frac{(1-\sec^2 x)}{1-\cos x} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 0}\frac{0-2\sec x(\sec x\tan x)}{} = \lim \left[-2\sec^2 x\tan x\right]$$

$$= \lim_{x \to 0} \frac{0 - 2\sec x\,(\sec x \tan x)}{0 - (-\sin x)} = \lim_{x \to 0} \left[\frac{-2\sec^2 x \tan x}{\sin x} \right]$$

$$= \lim_{x \to 0} \frac{-2 \frac{1}{\cos^2 x} \frac{\sin x}{\cos x}}{\sin x} = \lim_{x \to 0} \underbrace{-2\sec^3 x}$$

$$= -2$$

$\boxed{\text{OR}}$ $\lim_{x \to 0} \frac{-2\sec^2 x \left(\frac{\tan x}{x}\right) x}{\sin x}$ ← only when fun are in mul

$$= \lim_{x \to 0} -2\sec^2 x = -2$$

Que $\lim_{x \to 0} \frac{x e^x - \log(1+x)}{x^2}$ $\left(\frac{0}{0} \text{ form}\right)$ Use L'Hos Rule

$\log 1 = 0$

Sol: $\lim_{x \to 0} \frac{\underbrace{x e^x} + e^x - \frac{1}{1+x}}{2x}$ $\left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x \to 0} \frac{x e^x + e^x + e^x + \frac{1}{(1+x)^2}}{2} = \frac{2+1}{2} = \frac{3}{2}$$

Que Solve $\lim$ $e^x - e^{-x}$ ...

Que: Solve $\lim_{x\to 0} \dfrac{e^x - e^{-x} - 2\log(1+x)}{x \dfrac{\sin x}{x} x}$

Sol: $\lim_{x\to 0} \dfrac{e^x - e^{-x} - 2\log(1+x)}{x^2}$ $\left[\because \lim_{x\to 0} \dfrac{\sin x}{x} = 1\right]$

$\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x\to 0} \dfrac{e^x + e^{-x} - \dfrac{2}{1+x}}{2x}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x\to 0} \dfrac{e^x - e^{-x} + \dfrac{2}{(1+x)^2}}{2} = \dfrac{2}{2} = 1$

Que: $\lim_{x\to 0} \dfrac{\log x^2}{\cot x^2}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$\log 0 = -\infty$

$\cot 0 = \frac{1}{0} = \infty$

By L'Hospital Rule

$\lim_{x\to 0} \dfrac{\frac{1}{x^2}(2x)}{-\operatorname{cosec}^2 x^2 (2x)} = \lim_{x\to 0} \dfrac{-1/x^2}{\operatorname{cosec}^2 x^2}$

$= -\lim_{x\to 0} \dfrac{\sin^2 x^2}{x^2} \times \dfrac{x^2}{x^2}$

$x \to 0 \qquad x^2 \qquad x^2$

$$= -\lim_{x \to 0} \left(\frac{\sin^2 x^2}{x^4} \right) x^2$$

$$= -\lim_{x \to 0} \left(\frac{\sin x^2}{x^2} \right)^2 \cdot x^2$$

$$= -\lim_{x \to 0} x^2 \qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \right]$$

$$= 0$$

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$\frac{0}{0}$, $\frac{\infty}{\infty}$] → L'Hospital Rule

$$\lim_{x\to c} \frac{F(x)}{g(x)} = \lim_{x\to c} \frac{F'(x)}{g'(x)} = \cdots$$

Que: $\lim_{x\to 0} \frac{\log x^2}{\cot x^2} \to ?$

$\log 0 = -\infty$

$\cot 0 = \infty$

$\frac{\infty}{\infty}$ form, By L-H Rule,

$$\lim_{x\to 0} \frac{\frac{1}{x^2}\cdot \cancel{2x}}{-\operatorname{cosec}^2 x^2 \,\cancel{2x}} = \lim_{x\to 0}\left[-\frac{\sin^2 x^2}{x^2}\right]$$

$$= -\lim_{x\to 0}\left[\frac{\sin x^2}{x^2}\right]\sin x^2$$

$$\boxed{\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1}$$

$$= 1\cdot\sin 0 = 0$$

Que: $\lim_{x\to 0} \frac{x - \tan x}{x^3} = ?$

Put $x=0$

$\frac{0-\tan 0}{} = 0$

$x \to 0$ $\quad x^2$

$\frac{0}{0}$ form, By L-H Rule

$$\lim_{x \to 0} \left(\frac{1-\sec^2 x}{3x^2}\right) = \lim_{x\to 0} \frac{-\tan^2 x}{3x^2}$$

$$= \frac{-1}{3} \lim_{x \to 0} \left[\frac{\tan x}{x}\right]^2$$

$$= \frac{-1}{3}(1) = \frac{-1}{3}$$

$\frac{0 - \tan 0}{0} = \frac{0}{0}$

Recall

$\sec^2\theta - 1 = \tan^2\theta$

$\lim_{x\to 0} \frac{\tan x}{x} = 1$

II^nd Method

$$\lim_{x \to 0} \frac{1-\sec^2 x}{3x^2} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$\lim_{x \to 0} \frac{0 - 2\sec x\,(\sec x \tan x)}{6x}$$

$$\lim_{x \to 0} \frac{\frac{-1}{3} \frac{1}{\cos^2 x} \frac{\sin x}{\cos x}}{x}$$

$$= \lim_{x \to 0} \frac{-1}{3} \left(\frac{\sin x}{x}\right) \frac{1}{\cos^3 x} = \frac{-1}{3}$$

Que: $\lim_{x \to 0} \log_{\sin 2x}(\sin x)$

Recall

$\log_a b = \frac{\log b}{\log a}$

$\log 0 = -\infty$

Sol: $\lim_{x \to 0} \frac{\log(\sin x)}{\log(\sin 2x)}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

By L-H Rule

$$\lim_{x \to 0} \frac{\frac{1}{\sin x}\cos x}{\frac{1}{\sin 2x}\cos 2x \cdot 2} = \lim_{x \to 0} \frac{1}{2} \frac{\sin 2x \cos x}{\sin x \cos 2x}$$

$$= \lim_{x \to 0} \frac{1}{\cancel{2}} \frac{(\cancel{2} \cancel{\sin x} \cos x)\cos x}{\cancel{\sin x} \cos 2x}$$

$$= \frac{\cos^2 0}{\cos 0} = 1$$

Imp

Que: $\lim_{x \to \infty} \frac{x^n}{e^x} = ?$ where $n \in N$

Put $x = \infty$

$\frac{(\infty)^n}{e^\infty} = \frac{\infty}{\infty}$

$\frac{\infty}{\infty}$ form, By L-H Rule

Ist Time → $\lim_{x \to \infty} \frac{n x^{n-1}}{e^x}$

$\frac{\infty}{\infty}$ form, By L-H Rule

2nd Time → $\lim_{x \to \infty} \frac{n(n-1) x^{n-2}}{e^x}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

2^{nd} Time $\rightarrow \lim_{x \to \infty} \frac{n(n-1)x}{e^x}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

After applying L-H Rule 'n' times, we get

$$\lim_{x \to \infty} \frac{[n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1]\, x^{n-n}}{e^x}$$

$$\lim_{x \to \infty} \frac{n!\, x^0}{e^x} = n! \lim_{x \to \infty} \frac{1}{e^x} = n! \frac{1}{e^\infty}$$

$$= n! \left[\frac{1}{\infty}\right] = n!(0) = 0$$

Que. $\lim_{x \to \infty} \frac{x^n}{e^x}$, $n \in N$ equals to

$n = 1 \in N$

a) 1

b) 0

c) 1/2

d) one

$$\lim_{x \to \infty} \frac{x}{e^x} = \lim_{x \to \infty} \frac{1}{e^x} = \frac{1}{\infty} = 0$$

Que. $\lim_{x \to \infty} \frac{x + \sin x}{x + \cos x} = ?$

Sol: $$\lim_{x \to \infty} \frac{\cancel{x}\left[1 + \frac{\sin x}{x}\right]}{\cancel{x}\left[1 + \frac{\cos x}{x}\right]}$$

$$= \lim_{x\to\infty} \frac{\left[1 + \frac{\sin x}{x}\right]}{\left[1 + \frac{\cos x}{x}\right]}$$

$$= 1$$

$-1 \le \sin x \le 1$

$\lim_{x\to\infty} \frac{\text{finite}}{x} = 0$

If $\lim_{x\to c} F(x)\cdot g(x)$ and $\lim_{x\to c} g(x) = 0$, $F(x)$ is bounded

then $\lim_{x\to c} F(x)g(x) = 0$

$$\lim_{x\to\infty} (\sin x)\frac{1}{x} = 0$$

$\sin x \downarrow$ bdd

Que: $\lim_{x \to \frac{\pi}{2}} \left[\frac{\tan x}{\log(\cos x)} \right]$

Put $x = \frac{\pi}{2}$

$$\frac{\tan \pi/2}{\log\left(\cos\frac{\pi}{2}\right)} = \frac{\infty}{\log(0)} = \frac{\infty}{-\infty}$$

Sol: $\frac{\infty}{\infty}$ form, By L-H Rule,

$$\lim_{x \to \frac{\pi}{2}} \frac{\sec^2 x}{\frac{1}{\cos x}(-\sin x)}$$

$$= \lim_{x \to \frac{\pi}{2}} -\frac{1}{\cos^{\cancel{2}} x} \frac{\cancel{\cos x}}{\sin x}$$

$$= \lim_{x \to \pi/2} \frac{-1}{\sin x \cos x}$$

$$= \frac{-1}{(1)(0)} = -\infty$$

① $\frac{0}{0}, \frac{\infty}{\infty}$] By L-Hospital Rule

② $0 \cdot \infty$, $\infty - \infty$] How to solve ?

$\underline{0 \cdot \infty \text{ form}}$: First Reduce it to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ & then L'Hospital Rule

Que: $\underbrace{\lim_{x \to 0} x \log x}$ = ?

Put $x = 0$

$\log 0 = -\infty$

$0 \cdot \infty$ form

$0 \cdot \infty = \frac{0}{0}$ or $\frac{\infty}{\infty}$

$0 \cdot \infty = \frac{0}{1/\infty} = \frac{0}{0}$

$0 \cdot \infty = \frac{\infty}{1/0} = \frac{\infty}{\infty}$

Que: $\lim_{x \to 0} x^{(1)} (\log x)'$ ($0 \cdot \infty$ form)

Sol: $\lim_{x\to 0} \frac{\log x}{1/x}$ $\left(\frac{\infty}{\infty}\right)$

By L-H Rule,

$\lim_{x\to 0} \frac{1/x}{-1/x^2} = \lim_{x\to 0}(-x) = 0$

$\lim_{x\to 0} \frac{x}{1/\log x}$ $\left(\frac{0}{0}\right)$

By L-H Rule,

$\lim_{x\to 0} \frac{1}{-\left(\frac{1}{\log x}\right)^2 \cdot \frac{1}{x}}$

$= \lim_{x\to 0} \frac{x}{-\left(\frac{1}{\log x}\right)^2}$

we are not getting answer?

NOTE: If log is present then keep it in numerator

Qu: $\lim_{x\to 1} (1-x)\tan\frac{\pi x}{2} = ?$

(0 · ∞ form)

Put $x = 1$

$\tan\frac{\pi}{2} = \infty$

$0\cdot\infty$ form

Sol: $\lim_{x\to 1} \frac{1-x}{\cot\frac{\pi x}{2}}$ $\left(\frac{0}{0} \text{ form}\right)$

By L-H Rule

$= \lim_{x\to 1} \frac{\cancel{-}1}{\cancel{-}\operatorname{cosec}^2\frac{\pi x}{2}\cdot\frac{\pi}{2}} = \frac{1}{\operatorname{cosec}^2\frac{\pi}{2}\cdot\frac{\pi}{2}} = \frac{2}{\pi}$

Qu: $\lim_{x\to 0} x^m(\log x)^n$ where $m, n \in N = \{1, 2, \ldots\}$

Qn. $\lim_{x \to 0} x^m (\log x)^n$ where $m, n \in N - \{1, 2, \ldots\}$

Sol: $0 \cdot \infty$ form,

$$\lim_{x \to 0} \frac{(\log x)^n}{1/x^m} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

By L-H Rule, $\left[\frac{1}{x^m} = x^{-m}\right.$

$$\lim_{x \to 0} \frac{n(\log x)^{n-1}\left(\frac{1}{x}\right)}{-m\, x^{-m-1}}$$

$$\text{I}^{st} = \lim_{x \to 0} \frac{n(\log x)^{n-1}}{(-m)\, x^{-m}} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

By L-H Rule

$$\lim_{x \to 0} \frac{n(n-1)(\log x)^{n-2} \cdot \left(\frac{1}{x}\right)}{(-m)(-m)\, x^{-m-1}}$$

$$2^{nd} = \lim_{x \to 0} \frac{n(n-1)(\log x)^{n-2}}{(-m)^2\, x^{-m}} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

After applying L-H Rule, 'n' times,

$$\lim \frac{[n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1](\log x)^{n-n}}{}$$

$$\lim_{x \to 0} \frac{[n(n-1)(n-2) \text{-----} 3\cdot 2\cdot 1](\log x)^{-}}{(-m)^n \, x^{-m}}$$

$$\lim_{x \to 0} \frac{n! \, x^m}{(-m)^n} = 0$$

Qu: $\lim_{x \to 0} x^m (\log x)^n$ ✓Put $m = n = 1$

$$\lim_{x \to 0} x \log x = 0$$

$(\infty - \infty)$ form

$$\lim_{x \to c} [f(x) - g(x)] = \boxed{\infty - \infty}$$

How to convert it in $\left(\frac{0}{0}\right)$ form.

$$f(x) - g(x) = \frac{1}{1/f(x)} - \frac{1}{1/g(x)}$$

a b

$$\lim_{x \to c} [f(x) - g(x)] = \frac{1/g(x) - 1/f(x)}{\frac{1}{f(x)} \cdot \frac{1}{g(x)}} \quad \left(\frac{0}{0}\right) \text{ form.}$$

$f(x) \to \infty$, $g(x) \to \infty$

$\frac{1}{f(x)} \to 0$, $\frac{1}{g(x)} \to 0$

Que: $\underset{x\to\infty}{lt}\ \underbrace{x}\tan^{-1}\left(\frac{2}{x}\right) = ?$

Put $x=\infty$

$\tan^{-1}\left(\frac{2}{\infty}\right)$

$= \tan^{-1}(0)$

$= 0$

$\infty \cdot 0$ form

Sol: $\infty\cdot 0$ form

$$\underset{x\to\infty}{lt}\ \frac{\tan^{-1}\left(\frac{2}{x}\right)}{1/x}\quad\left(\frac{0}{0}\text{ form}\right)$$

By L-H Rule

$$\underset{x\to\infty}{lt}\ \frac{\frac{1}{1+\left(\frac{2}{x}\right)^2}\left[\frac{-2}{\cancel{x^2}}\right]}{-1/\cancel{x^2}}$$

$$= \underset{x\to\infty}{lt}\ 2\left[\frac{x^2+4-4}{x^2+4}\right] = 2\left[1-\frac{4}{x^2+4}\right]$$

$$= 2\left[1-\frac{1}{\infty}\right]$$

$$= 2[1-0]$$

$$= 2$$

Que: $\underset{x\to 0}{lt}\left[\underbrace{\operatorname{cosec} x}_{F(x)} - \underbrace{\frac{1}{x}}_{g(x)}\right]$

$$\underset{x\to c}{lt}\ f(x)-g(x) = \frac{\frac{1}{g(x)}-\frac{1}{f(x)}}{\frac{1}{f(x)\,g(x)}}$$

Sol: $= \underset{x\to 0}{lt}\ \frac{\frac{1}{1/x}-\frac{1}{\operatorname{cosec}x}}{1} = \underset{x\to 0}{lt}\ \frac{x-\sin x}{x.(\sin x)}$

$$\lim_{x\to0} \frac{\ \cdots\ }{\frac{1}{1/x \ \operatorname{cosec}x}} = \lim_{x\to0} \frac{\ \cdots\ }{x(\sin x)} \quad \left(\frac{0}{0} \text{ form}\right)$$

By L-H Rule,

$$\lim_{x\to0} \frac{1-\cos x}{\underline{x\cos x} + \sin x} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x\to0} \frac{\sin x}{-x\sin x + \cos x + \cos x} = \frac{0}{0+1+1} = 0$$

Que: $\lim_{x\to\frac{\pi}{2}} (\sec x - \tan x) = ?$

Sol: $\infty - \infty$ form

$$\lim_{x\to\frac{\pi}{2}} \left[\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right] = \lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos x}$$

$\frac{0}{0}$ form, By L-H Rule,

$$\lim_{x\to\frac{\pi}{2}} \frac{\cancel{-}\cos x}{\cancel{-}\sin x} = \frac{\cos \pi/2}{\sin \pi/2} = \frac{0}{1} = \boxed{0}$$

Que: $\lim \left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right] = ?$

Que: $\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x} \right] = ?$

Sol: $\infty - \infty$ form

$$\lim_{x \to 0} \frac{\sin^2 x - x^2}{x^2 \left[\frac{\sin^2 x}{x^2}\right] x^2}$$

$\left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right.$

$$= \lim_{x \to 0} \frac{\sin^2 x - x^2}{x^4} \quad \left(\frac{0}{0} \text{ form}\right)$$

By L-H Rule,

$$= \lim_{x \to 0} \frac{(2\sin x \cos x) - 2x}{4x^3}$$

$$= \lim_{x \to 0} \frac{\sin 2x - 2x}{4x^3} \quad \left(\frac{0}{0} \text{ form}\right)$$

By L-H Rule

$$= \lim_{x \to 0} \frac{\cos 2x\,(2) - 2}{12x^2}$$

$$= \frac{2}{12} \lim_{x \to 0} \frac{\cos 2x - 1}{x^2} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \frac{1}{6} \lim_{x \to 0} \frac{-\sin 2x\,(2)}{2x}$$

$$= \frac{-2}{6} \lim \left(\frac{\sin 2x}{2x} \right) = \frac{-1}{3}(1) = \frac{-1}{3}$$

$$= \frac{-2}{6} \lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right) = \frac{-1}{3}(1) = \frac{-1}{3}$$

Qu: $\lim_{x \to 0} \left[\frac{1}{e^x - 1} - \frac{1}{x}\right]$

Sol: $\lim_{x \to 0} \left[\frac{x - e^x + 1}{x(e^x - 1)}\right]$ $\left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x \to 0} \left[\frac{1 - e^x}{x(e^x) + (e^x - 1)}\right] \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 0} \left[\frac{-e^x}{xe^x + e^x + e^x}\right] = \frac{-1}{2}$$

$0^0, 1^\infty, \infty^0$ forms

$\lim_{x\to c} [F(x)]^{g(x)}$ — ?

Let $y = [F(x)]^{g(x)}$

Take log

$\log y = \log [F(x)]^{g(x)}$

$\rightarrow \log y = g(x) \log F(x) = l \rightarrow$ $\boxed{0\cdot\infty \text{ Form}}$

Solve it by converting it into $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$\boxed{\lim_{x\to c} y = e^l}$

$0^0 \rightarrow \log 0^0 = 0 \log 0 = 0\cdot\infty$

$1^\infty \rightarrow \log 1^\infty = \infty \log 1 = 0\cdot\infty$

$\infty^0 \rightarrow \log \infty^0 = 0\cdot\infty$

Qu: $\lim_{x\to 0} x^x = ?$

Sol: 0^0 form

Let $y = x^x$

Take log on both sides

$\log y = \log x^x$

$\log y = x \log x$ $\quad [\because \log a^b = b \log a$

Take limit on both sides

Put $x = 0$

$x^x = 0^0$

$\lim \log y = \lim x \log x$ $\quad [\ldots]$

Take limit on both sides

$$\lim_{x\to 0} \log y = \lim_{x\to 0} x \log x \quad [0\cdot\infty \text{ form}]$$

$$\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{\log x}{1/x} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

By L-H Rule,

$$\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{1/x}{-1/x^2} = \lim_{x\to 0} (-x) = 0$$

$$\lim_{x\to 0} \log y = 0$$

$$\Rightarrow \lim_{x\to 0} y = e^0 = 1$$

If $\log y = x$ then $y = e^x$

Que: $\lim_{x\to 1} (x)^{\frac{1}{x-1}} = ?$

Sol: 1^∞ form

Put $x = 1$

$(x)^{\frac{1}{x-1}} = (1)^{\frac{1}{0}}$
$= 1^\infty$

Let $y = (x)^{\frac{1}{x-1}}$

Take log $\Rightarrow \log y = \log(x)^{\frac{1}{x-1}}$

$$\log y = \frac{1}{x-1} \log x$$

Apply limit $x \to 1$

$$\lim_{x\to 1} \log y = \lim_{x\to 1} \frac{\log x}{x-1} \quad \left(\frac{0}{0} \text{ form}\right)$$

Apply L-H Rule,

$$\lim_{x\to 1} \log y = \lim_{x\to 1} \frac{1/x}{1} = \lim_{x\to 1} \frac{1}{x} = 1$$

$$\Rightarrow \lim_{x\to 1} \log y = 1 \quad \Rightarrow \lim_{x\to 1} y = e^{1} = e$$

Que: $\lim_{x\to 0} (\operatorname{cosec}x)^{\frac{1}{\log x}}$

Put $x=0$

$\operatorname{cosec}0 = \infty$

$\log 0 = -\infty$

$\frac{1}{\log 0} = \frac{1}{-\infty} = 0$

Sol: ∞^0 form

Let $y = (\operatorname{cosec}x)^{\frac{1}{\log x}}$

Take log

$$\log y = \frac{1}{\log x} \log \operatorname{cosec}x$$

Take limit $x\to 0$

$$\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{\log \operatorname{cosec}x}{\log x} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

$\boxed{\log\infty = \infty}$

By L-H Rule,

$$\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{\frac{1}{\cancel{\operatorname{cosec}x}} \cdot -\cancel{\operatorname{cosec}x}\cot x}{1/x}$$

$$\lim_{x\to 0} \log y = -\lim_{x\to 0} x\underbrace{\cot x} = -\lim_{x\to 0} \frac{x}{\tan x} = -1$$

$$\Rightarrow \lim_{x\to 0} \log y = -1$$

$$\Rightarrow \lim_{x\to 0} y = e^{-1} = \frac{1}{e}$$

$$\Rightarrow \lim_{x\to 0} y = e^{-1} = \frac{1}{e}$$

$$0^{\infty} - ?$$

$$\log 0^{\infty} = \infty \log 0 = \infty(-\infty)$$
$$= \underbrace{\infty}$$

Qui: Prove $\lim_{x\to\infty} (1+x)^{\frac{1}{x}} = 1$

∞^0 form [Put $x=\infty$

Sol: ∞^0 form

Take $y = (1+x)^{\frac{1}{x}}$

Put $x=\infty$

$(1+\infty)^{\frac{1}{\infty}} = (\infty)^0$

Take log, $\log y = \frac{1}{x}\log(1+x)$

Take limit $x \to \infty$

$$\lim_{x\to\infty} \log y = \lim_{x\to\infty} \frac{\log(1+x)}{x} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

By L-H Rule,

$$\lim_{x\to\infty} \log y = \lim_{x\to\infty} \frac{\frac{1}{1+x}}{1}$$

$$= \lim_{x\to\infty} \frac{1}{1+x} = \frac{1}{1+\infty}$$

$$\lim_{x\to\infty} \log y = \frac{1}{\infty} = 0$$

$$\Rightarrow \lim_{x\to\infty} \log y = 0$$

$$\Rightarrow \lim_{x\to\infty} y = e^0 = 1$$

Que: Prove $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

Que: $\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}} = ?$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$(1)^\infty$ form

$\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$

$$y = \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$$

(1^∞) form

$$\log y = \frac{1}{x^2}\log\left(\frac{\sin x}{x}\right)$$

Limit $x \to 0$, $\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{\log\left(\frac{\sin x}{x}\right)}{x^2}$ $\left(\frac{0}{0}\text{ form}\right)$

By L-H Rule

$$= \lim_{x\to 0} \frac{\frac{1}{\left(\frac{\sin x}{x}\right)}\left[\frac{x\cos x - \sin x}{x^2}\right]}{2x}$$

$$\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{x\cos x - \sin x}{2x^3} \quad \left[\frac{0}{0}\text{ form}\right]$$

By L-H Rule

$$= \lim_{x\to 0} \frac{-x\sin x + \cos x - \cos x}{6x^2}$$

$$= \lim_{x\to 0} \frac{-x\sin x}{6x^2} = \lim_{x\to 0} \frac{-1}{6}\frac{\sin x}{x}$$

$$\lim_{x\to 0} \log y = \frac{-1}{6}\left[\lim_{x\to 0} \frac{\sin x}{x}\right] = \frac{-1}{6}$$

$$\Rightarrow \lim_{x\to 0} y = e^{-1/6}$$